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AN ANALYSIS OF MEMBRANE ACTION IN ONE-WAY CONCRETE MEMBERS EXTERNALLY BONDED WITH FRP

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ABSTRACT

A theoretical investigation of membrane actions including compressive membrane action (CMA) and tensile membrane action (TMA) in one-way concrete members (i.e. beams and slabs) externally strengthened with Fiber Reinforced Polymer (FRP) is conducted in this paper. The favourable properties of membrane actions and its predictive modelling are first presented. An extended Park and Gamble's method is proposed to account for the CMA effect and a spring based model is selected to consider the effect of TMA. By defining an enhancement factor, the study focuses on how CMA and TMA affect the load bearing capacities of the members considered with material parameters such as the FRP reinforcement area and steel reinforcement area. The results show that the effect of CMA on the load bearing capacity is more pronounced in beam members than in slab members, whereas the opposite applies to the enhancement factors of TMA between beams and slabs. A comparison of membrane action between conventional reinforced concrete (RC) members and FRP-strengthened RC members is given as well. Finally, remarks on how to make use of the favourable properties of membrane actions are presented.

1 INTRODUCTION

Since membrane action was first experimentally investigated by Westergaard and Slater [1] by examining the membrane behaviour in numerous full-scale tests on laterally restrained floor panels in 1920s, the phenomenon of membrane behaviour in reinforced concrete (RC) structures was introduced, developed and used in an extensive way. On one hand, the compressive membrane action (CMA), which occurs at relatively small displacements in laterally restrained RC members, has been given much attention in research particularly in RC slabs due to its practical importance for performance-based design applications [2-3]. Research results have shown that CMA is beneficial in strength enhancement. With regard to the investigation of CMA in concrete members, one main method, proposed by Park & Gamble [4], is using the plastic theory to obtain the member's resistance under CMA by considering strain compatibility and force equilibrium at the sectional level based on a perfectly rigid plastic model. This method considers CMA to be initially associated with the bending capacity and is approperiate for estimating the CMA capacity of laterally restrained RC members. On the other hand, the tensile membrane action (TMA) or catenary action, which occurs at very large displacements in RC members with horizontal restraints, has also obtained much attention in experimental and numerical investigations and has been proven to be useful to improve the robustness of structures subjected to accidental scenarios [5-6]. In contrast to CMA, TMA in one-way RC members can only be developed at a stage of large deformations when an advanced state of cracking and concrete damage has occured, finally leaving only the reinforcement with high ductility (e.g. steel bars) to act as a tensile component.

With the increased application of Fiber Reinforced Polymer (FRP) for the strengthening of concrete structures, much work has been done to investigate the effect of FRP on the strength enhancement in comparison to conventional concrete structures [7]. Accordingly, regarding the concept of membrane actions, it is desirable to investigate how these actions are affected when FRP reinforcement is taken into account.

In this paper, the framework of Park and Gamble's method to analyze CMA in conventional RC members is first extended to account for the effect of FRP on the load bearing capacity of RC members considering CMA, and a spring based model is adopted to account for TMA in the RC members studied. The effects of CMA and TMA on the load bearing capacities of the RC members were investigated in a parametrical study by defining an enhancement factor. Material parameters such as the FRP reinforcement area and steel reinforcement area are considered in this research. A comparison of membrane action between conventional RC members and FRP-strengthened RC members is given as well. Further, remarks on how to make use of the favourable properties of membrane actions are presented.

2 PREDICTION MODELS

2.1 General assumptions

The membrane actions investigated in this paper are analyzed by performing analysis of free bodies and sectional analysis of critical sections. For the purpose of generality, to account for the CMA and TMA for both conventional one-way concrete members and those strengthened with FRP and as inspired by Park and Gamble's method, a model with four idealized plastic hinges formed symmetrically along the member considered is chosen (see Figure 1a). A perfectly rigid plastic mechanism is basically assumed. A complete symmetry along the span is assumed with respect to geometry, reinforcement, loading, boundary conditions and deformations. The lateral restraints are idealized to be equivalent axial springs with stiffness K_a. Figure 1a shows the schematic view of the model, where β is the ratio of the span length from the plastic hinge at the member end to the nearest hinge in the span, l_n, to the member span, l.

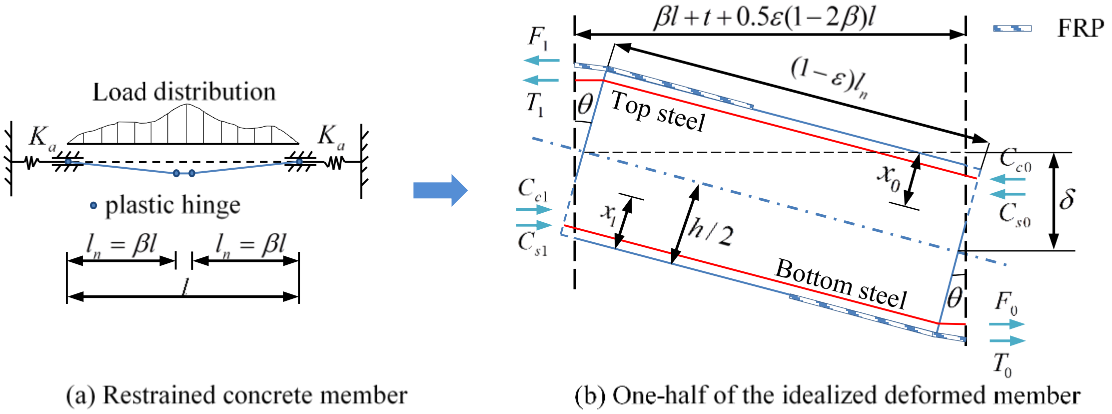


Figure 1 Schematic view of a restrained concrete member and one-half of its deformed shape

To calculate the sectional moments and forces, widely-adopted assumptions are used, including the assumptions of plane-section, an idealized equivalent rectangular stress block for compressive concrete, neglect of concrete tensile strength and a bilinear stress-strain relationship for steel bars. In cases where FRP is applied, a full composite action of FRP, a linear stress-strain relationship for FRP and the neglecting of the compressive strength of FRP are also assumed as proposed by the *fib* guidelines [8]. More specifically, for the sake of simplicity, it is assumed that under CMA conditions the FRP is not debonding, while under TMA conditions (at very large deformations) the FRP is no longer active.

2.2 CMA model

Due to the symmetrical property of the model shown in Figure 1a, it is feasible to investigate only one-half of model under deformation, as shown in Figure 1b. The force equilibrium requires that the resultant of horizontal forces at both considered sections (0: midspan; 1: member end) equals the acting axial force N, then the following equation can be obtained

$$N = C_{c0} + C_{s0} - T_0 - F_0 = C_{c1} + C_{s1} - T_1 - F_1 \tag{1}$$

where C_{c0} and C_{c1} are the concrete compressive forces, C_{s0} and C_{s1} the steel compressive forces, T_0 and T_1 the steel tensile forces, and F_0 and F_1 the FRP tensile forces, acting on sections at the span and the end, respectively; and N is the axial force which is related to the axial deformation ability of the member considered. In this paper, the compressive strain of the beam is assumed to be uniformly distributed over the longitudinal direction, i.e. $\varepsilon = N/(E_c A_c)$, where E_c is elastic modulus of concrete and A_c is the cross-sectional area. Apparently, the contraction of the middle portion shown in Figure 1a is $(1\text{-}2\beta)\varepsilon l$. Further, for a given deflection in the span δ, we have $\tan\theta \approx \delta/\beta l$. This rotation θ is so small that $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ can be assumed. Therefore, the compatibility requirement can be expressed as in Equation (2)

$$h - x_0 - x_1 = \frac{\delta}{2} + \frac{N\beta l^2}{2\delta}\left(\frac{1}{E_c A_c} + \frac{2}{K_a l}\right) \tag{2}$$

where x_0, x_1 are the neutral-axis depths at the span and ends sections of the member, respectively, and h is the member's total depth.

It can be seen that, for a given δ, the only unknowns in the above equations are x_1 and x_0, which can be obtained by solving the Equations (1) and (2) simultaneously. Further, the load bearing capacity considering CMA can be calculated. Repeating such procedures, a maximum of a series of values of the load bearing capacity can be found and seen as the ultimate load bearing capacity considering CMA. In this paper, the ultimate concrete strain is 3.5‰ [8], the steel yield strain is calculated by its yield stress, and the ultimate strain of FRP is the test value or a value of 1.5% is selected if not specified [9]. In addition, the lateral stiffness (K_a) should be calculated considering the real structural system or, for simplicity, a relatively large stiffness, such as 1×10^6 kN/m can be selected if no such information can be obtained.

2.3 TMA model

As aforementioned, TMA is developed in one-way RC members at a stage of large deformations when an advanced state of cracking and concrete damage have occurred, finally leaving only the reinforcement with high ductility (e.g. steel bars) to act as a tensile net. Due to the large deformation capacity of steel reinforcement, it can be considered that in the TMA stage only the steel reinforcement acts as the load bearing component which acts as a tensile component fixed at the supports at the ends. Therefore, the load bearing capacity of the RC members considered in TMA can be calculated as given in Equation (3)

$$P = 2 f_y (A_{s0} + A_{s1}) \sin\theta \tag{3}$$

where A_{s0} and A_{s1} are the tensile reinforcement areas at the span and ends of the RC member, and f_y is the yield stress of the steel reinforcement. It is interesting to point out that, in case the RC member considered is clamped at both ends, it is possible that the resisting moments at the member's ends still exist when TMA starts to work. When the characteristic of interest of the member is the ultimate load bearing capacity, however, the effect of such moments is negligible because the concrete in tension is severely cracked and damaged and both the steel reinforcements at the top and bottom of the member are in tension. This means that, when the ultimate load bearing capacity considering TMA is reached, the demand of the tensile strain is quite large and the contact between concrete and longitudinal reinforcement is quite weak. Therefore, it is believed that the applied FRP has already failed when the ultimate load bearing capacity considering TMA is reached, either due to its low ultimate strain or its weakness with respect to debonding. With respect to the rotation θ, in general a corresponding deflection ranging from 7% to 10% of the member length is chosen [10]. For simplicity, 10% is chosen in this paper.

2.4 Model verification

In the previous paper of the authors [9], the CMA model was verified using three types of concrete beams: conventional RC beams, RC beams strengthened with FRP at sagging zones and RC beams strengthened with FRP at both sagging and hogging zones. The efficiency of the model to depict the behavior of CMA is proven. For more information, the reader is referred to [9].

Compared to the verification of CMA, the verification of the TMA is easier as the FRP reinforcement is not involved anymore. Test results of several authors such as Botte et al [6] can be adopted. In case of Slab 1 in [6], two layers of 16 ϕ10 steel bars (f_y = 555 MPa) were configured in this 8 m long slab. A maximum rotation of 11% was observed. According to Equation (3) the predicted load bearing capacity in TMA stage of this slab is 306.2 kN, with a 4% underestimation compared to the tested result of 320 kN. The underestimation of the predicted capacity is due to the hardening behavior of the steel reinforcement. Note that, however, a conservative model for TMA (using yielding strength but not hardening strength in Equation (3)) is desirable because in TMA stage the considered member is severely damaged and any overestimation would lead to unwanted collapse.

3 MEMBRANE ACTIONS IN ONE-WAY CONCRETE MEMBERS

As shown above, membrane actions are affected by many influencing factors, therefore it is necessary to select a non-dimension quantity to examine how the input properties affect the load bearing capacity when membrane actions are considered. It is straightforward to define an enhancement factor, α_P, to be the ratio of the load bearing capacity of the considered member taking CMA or TMA into account to that of a reference situation of the same member without consideration of CMA or TMA. E.g. for the case of an FRP strengthened RC beam, the enhancement factor can be defined as follows:

$$\alpha_{\mathrm{P},CMA,FRP} = \frac{P_{CMA,FRP}}{P_{0,FRP}} \tag{4}$$

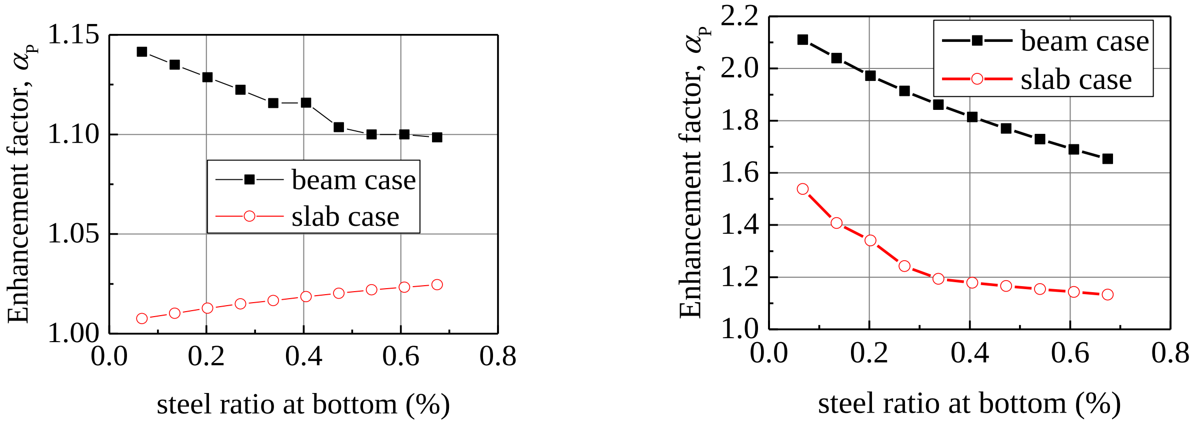
where $\alpha_{\mathrm{P,CMA,FRP}}$ is the enhancement factor quantifying the CMA effect, $P_{\mathrm{CMA,FRP}}$ is the beam peak resistances considering CMA and FRP strengthening calculated by the extensive method mentioned above, and $P_{\mathrm{0,FRP}}$ is the beam peak resistance calculated by *fib* bulletin 14 [8] of the same beam. Apparently, Equation (4) also applies for conventional RC beams. Similar rules apply for slabs.

In case of beams, a beam similar to the four-point loaded two-span beam in [11] is adopted here as the benchmark beam in the parametric study. This 10 m beam is assumed to be laterally restrained (K_a = 10^6 kN/m) at both ends with a 200 × 400 mm section and a value of 0.20 for β is selected. C30/37 concrete is used and three steel bars are initially placed along the beam length at both the beam top (2ϕ12+1ϕ18, A_{top} = 480.4 mm^2) and the beam bottom (2ϕ12+1ϕ20, A_{bottom} = 540.1 mm^2). The elastic modulus and the yield stress of the steel bars are 200 GPa and 500 MPa, respectively. In cases of FRP strengthening, at maximum a CFRP layer with a section area of 120 mm^2 ($A_{\mathrm{f,max}}$) is applied along the beam at tension zones and the elastic modulus and the ultimate strain of the FRP are considered 190 GPa and 1.5%, respectively. Unless stated otherwise, all these values further apply in this paper. In order to compare the CMA effects between beam elements and slab elements, the same reinforcement ratio (including steel and fiber reinforcement) is chosen for both beam and slab elements. For example, for a certain comparison, the ratio of the tensile reinforcement in a beam ($A_{\mathrm{s0,b}}/A_{\mathrm{c,b}}$) equals the ratio of the tensile reinforcement in a slab ($A_{\mathrm{s0,s}}/A_{\mathrm{c,s}}$).

3.1 CMA in one-way concrete members

The comparison of CMA effects in beam and slab systems was made with ($A_{\mathrm{f,max}}$) or without consideration of FRP reinforcement, as shown in Figure 2. In Table 1, the enhancement factors are provided both for 100% and 40% of $A_{\mathrm{f,max}}$.

In Figure 2 and Table 1, the steel reinforcement at the top is fixed. It can be easily seen from either Figure 2a or 2b that for a same configuration of tensile reinforcement the CMA effect capacity is more significant in enhancing the load bearing capacity in beam systems than in slab systems. In the latter case, the effect of CMA on improving the load bearing capacity is negligible, which can also be observed in Table 1.



(a) Strengthened with FRP ($A_{f,max}$) (b) No FRP is applied

Figure 2 CMA in one-way concrete systems

Another observation from Figure 2a and 2b (also Table 1) is that the FRP reinforcement leads to a decrease of the strength enhancement of CMA. This is due to the more pronounced influence of FRP strengthening in the reference calculation (without considering CMA), compared to the CMA calculation. For example, the value of α_P in Figure 2a is 1.14 while the value of α_P in Figure 2b is 2.04 for a same beam member with steel ratio at bottom of 0.135%. This observation also holds for slab members.

Table 1 Enhancement factor α_P of CMA in beam and slab systems

Item	Steel ratio (%)	0.068	0.203	0.338	0.473	0.608	0.675
beam	$A_f/A_{f,max}$ = 0.4	1.28	1.36	1.36	1.35	1.34	1.33
	$A_f/A_{f,max}$ = 1.0	1.14	1.13	1.12	1.10	1.10	1.10
slab	$A_f/A_{f,max}$ = 0.4	1.03	1.03	1.03	1.02	1.02	1.03
	$A_f/A_{f,max}$ = 1.0	1.01	1.01	1.02	1.02	1.02	1.02

3.2 TMA in one-way concrete members

To investigate TMA in one-way beam and slab systems, the same configurations of concrete members as in Section 3.1 is adopted again. Following Equation (3) and (4) in which the load bearing capacity considering CMA is replaced by that considering TMA, the results are shown in Table 2.

Table 2 Enhancement factor α_P of TMA in beam and slab systems

Item	Steel ratio (%)	0.068	0.203	0.338	0.473	0.608	0.675
beam	$A_f/A_{f,max}$ = 1.0	0.51	0.57	0.63	0.67	0.82	0.74
	$A_f/A_{f,max}$ = 0.0	1.45	1.46	1.46	1.47	1.47	1.47
slab	$A_f/A_{f,max}$ = 1.0	2.13	2.49	2.83	3.16	3.46	3.61
	$A_f/A_{f,max}$ = 0.0	6.22	6.57	6.85	7.08	7.28	7.37

In contrast to the difference in CMA effect between beam and slab systems, Table 2 shows that TMA is much more favorable in slab systems than in beam systems. In addition, Table 2 shows that in general the effectiveness of TMA enhancement increase with increasing steel reinforcement. It is interesting to note that for beams strengthened with an FRP reinforcement area of 120 mm^2, the enhancement factors are less than 1, which implies that for the considered FRP-strengthened beam systems it is not favourable to consider TMA (TMA enhancement of RC beam, with FRP no longer active, is smaller than FRP strengthening effect without considering TMA).

4 REMARKS AND CONCLUSIONS

Membrane actions in one-way concrete members externally bonded with FRP are examined by adopting the extended Park and Gamble's model and spring based model to take CMA and TMA into account, respectively, by defining an enhancement factor to quantify such effects. The analysis results show that CMA is more favourable in improving the load bearing capacity of beam systems than in slab systems, and is more advantageous in conventional one-way RC members than in FRP-strengthened one-way RC members. In the design process, the beneficial effect of CMA can be incorporated, especially for conventional RC beams, during the verification of ultimate limit state to realize a more optimized design.

In contrast to CMA in one-way RC members, however, TMA is more advantageous in slabs than in beams and is more significant in conventional RC members than in FRP-strengthened RC members. This implies it is feasible to expect the development of TMA to sustain accidental loads in conventional members with lateral stiffness at its ends. In FRP strengthened members at such large deformations, and assuming that loss of FRP has occurred under the accidental situation, the development of TMA to sustain the (higher) accidental loads becomes less obvious, especially in the case of beams.

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